

DOCUMENT RESUME

ED 277 746

TM 870 025

TITLE Mathematics, Grade 7. TEAMS Instructional Strategies Guide.

INSTITUTION Texas Education Agency, Austin.

PUB DATE Aug 86

NOTE 67p.; For related documents, see TM 870 022-039.

AVAILABLE FROM Texas Education Agency, William B. Travis Building, 1701 North Congress Avenue, Austin, TX 78701.

PUB TYPE Guides - Classroom Use - Guides (For Teachers) (052)

EDRS PRICE MF01/PC03 Plus Postage.

DESCRIPTORS Algebra; Basic Skills; *Behavioral Objectives; Computation; Elementary School Mathematics; Geometry; *Grade 7; Instructional Development; Junior High Schools; Learning Activities; *Learning Strategies; *Mathematics Instruction; Mathematics Skills; *Mathematics Tests; *Minimum Competency Testing; Multiple Choice Tests; Number Concepts; Probability; Problem Solving; State Programs; Teaching Guides; Testing Programs; Test Items

IDENTIFIERS Test Content; *Texas Educational Assessment of Minimum Skills

ABSTRACT

The Texas Educational Assessment of Minimum Skills (TEAMS) was developed by the Texas Education Agency (TEA), in response to Texas House Bill 72 (1984), which mandated a new basic skills assessment program for mathematics, reading, and writing at grades 1, 3, 5, 7, 9 and 11. This is one in a series of Instructional Strategies Guides designed to familiarize Texas educators with the types of strategies employed in assessing learner mastery of TEAMS objectives and to provide suggested instructional activities that can be used to promote mastery of those objectives. This guide focuses on the TEAMS mathematics objectives for grade 7. It provides detailed description of the test items used to measure each mathematics objective. The 11 objectives are subsumed under seven skill areas: (1) number concepts (decimals and percents); (2) computation (addition and subtraction of fractions, addition, subtraction, and multiplication of decimals); (3) problem solving (two-step word problems using whole numbers, decimals); (4) measurement (metric and customary measurement conversions); (5) geometry (geometric terms and figures, perimeter of polygons); (6) probability, statistics, and graphing (graphs and charts, possible outcomes); and (7) algebraic concepts (equations). The following information is provided for each objective: (1) a description of the objectives; (2) a sample item; (3) test questions; (4) answer choices; (5) answer analysis of the sample item; (6) instructional emphases; and (7) possible instructional activities. (JAZ)

* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *

ED277746



INSTRUCTIONAL STRATEGIES GUIDE

MATHEMATICS GRADE 7

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION
CENTER (ERIC)

☒ This document has been reproduced as
received from the person or organization
originating it.

☐ Minor changes have been made to improve
reproduction quality.

• Points of view or opinions stated in this docu-
ment do not necessarily represent official
OERI position or policy.

"PERMISSION TO REPRODUCE THIS
MATERIAL HAS BEEN GRANTED BY

L. Kemp

TO THE EDUCATIONAL RESOURCES
INFORMATION CENTER (ERIC)."

TEXAS EDUCATION AGENCY
William B. Travis Building
1701 North Congress Avenue
Austin, Texas 78701

August 1986

TM 870 025

TEAMS INSTRUCTIONAL STRATEGIES GUIDE

MATHEMATICS

GRADE 7

This Texas Education Agency publication is not copyrighted, and any or all sections may be duplicated. After an initial free distribution to authorized institutions, additional copies may be purchased for \$2.00 from the Publications Distribution Center.

TEXAS EDUCATION AGENCY
William B. Travis Building
1701 North Congress Avenue
Austin, Texas 78701

TABLE OF CONTENTS

	<u>Page</u>
Acknowledgments	ii
Introduction	1
Components of the Instructional Strategies Guide	1
Use of the Instructional Strategies Guide	3
General Nature of the Test Items	3
Skill Area: Number Concepts	
Objective 1: Convert decimals or percents to equivalent forms.	4
Skill Area: Computation	
Objective 2: Add and subtract fractions with like or unlike denominators.	9
Objective 3: Add, subtract, and multiply decimals.	16
Skill Area: Problem Solving	
Objective 4: Solve two-step word problems using the basic operations with whole numbers.	21
Objective 5: Solve two-step word problems using addition, subtraction, and multiplication with decimals.	27
Skill Area: Measurement	
Objective 6: Convert measurements within either the metric or customary system of measurement.	31
Skill Area: Geometry	
Objective 7: Identify common geometric terms and figures.	36
Objective 8: Find the perimeter of polygons.	40
Skill Area: Probability, Statistics, and Graphing	
Objective 9: Use information from charts or graphs to solve word problems.	46
Objective 10: Determine the number of possible outcomes for a given situation.	53
Skill Area: Algebraic Concepts	
Objective 11: Solve one-step equations.	57

ACKNOWLEDGMENTS

Under contract with the Texas Education Agency, the Information Services Division of National Computer Systems subcontracted with IOX Assessment Associates to develop this TEAMS instructional strategies guide.

Key reviewers of early drafts of the guide were Lisa Chandler, Norlene M. Kunkel, Barbara Montalto, Cathy Seeley Peavler, and Patricia Sachse Porter of the Texas Education Agency.

INTRODUCTION

During 1980-1985 the Texas Education Agency (TEA) administered a criterion-referenced testing program, the Texas Assessment of Basic Skills (TABS). That program was designed to measure minimum skills in mathematics, reading, and writing at Grades 3, 5, and 9. In July 1984, under the provisions of House Bill 72, a new basic skills assessment program was mandated in mathematics, reading, and writing at Grades 1, 3, 5, 7, 9, and 11. In response, development of the Texas Educational Assessment of Minimum Skills (TEAMS) was instituted by TEA during the 1985-1986 school year.

School district personnel from across Texas participated in all aspects of the development of TEAMS. Basic skills objectives were selected after a thorough review of the State Board of Education Rules for Curriculum, GE4 450 01 (Essential Elements). Under TEA supervision measurement specifications and test items were developed for each objective. Statewide field testing was conducted to ensure the quality of test items. The State Board of Education then approved the TEAMS tests for administration.

Because of the significance of the TEAMS program, TEA is providing Texas educators with instructional strategies guides for all objectives measured on each TEAMS examination. These guides describe the TEAMS objectives as well as instructional considerations and activities that can be used to promote mastery of TEAMS objectives.

This instructional strategies guide focuses on the TEAMS mathematics objectives for Grade 7. It provides a detailed description of the test items used to measure each mathematics objective. In addition, a brief instructional analysis is provided for each objective as are instructional activities that can help students master the objective.

Texas educators will undoubtedly vary in the ways they wish to promote learner mastery of the TEAMS objectives. Thus, the instructional components of this guide are not intended to be prescriptive. Teachers should view suggested activities as potential approaches that lead to mastery of the TEAMS objectives.

Components of the Instructional Strategies Guide

Eleven TEAMS mathematics objectives are measured at the seventh grade level. These objectives are subsumed under one of seven skill areas. The distribution of mathematics objectives by skill area is shown on the next page.

<u>SKILL AREA</u>	<u>OBJECTIVE</u>
<u>Number Concepts</u>	Decimals and Percents
<u>Computation</u>	Addition and Subtraction of Fractions
	Addition, Subtraction, and Multiplication of Decimals
<u>Problem Solving</u>	Whole Numbers Decimals
<u>Measurement</u>	Metric and Customary Measurement Conversions
<u>Geometry</u>	Geometric Terms and Figures Perimeter of Polygons
<u>Probability, Statistics, and Graphing</u>	Graphs and Charts Possible Outcomes
<u>Algebraic Concepts</u>	Equations

This guide provides the following information for each of the mathematics objectives included in the TEAMS Grade 7 test.

Description of the objective. This section provides a succinct overview of the objective's focus.

Sample item. An illustrative sample item follows the description of each objective. This item is consistent with the test specifications for the objective, hence is representative of the TEAMS items used to measure students' mastery of the objective.

Test questions. To provide a more detailed view of how each TEAMS objective is assessed, the next section provides a description of the test question. All of the Grade 7 mathematics items present a stimulus that precedes four answer choices.

Answer choices. The next section deals with the response to each multiple-choice test item. Important characteristics of the answer choices are described.

Answer analysis for sample item. An analysis of the answer choices from the sample item is then provided. This analysis explains why the correct answer is right and why the incorrect answers are wrong.

Instructional emphases. Considerations relevant to the design of instructional sequences for each TEAMS objective are described in this section. These instructional analyses are not intended to represent the only important points associated with instruction aimed at a particular objective. However, it is hoped that these discussions will stimulate educators' consideration of each objective.

Possible instructional activities. A final section discusses several instructional activities for each objective that Texas educators may consider as they organize their instructional plans.

Use of the Instructional Strategies Guide

This TEAMS instructional strategies guide is designed to accomplish two goals. First, the guide enables Texas educators to become familiar with the types of strategies employed to assess learner mastery of the TEAMS Grade 7 mathematics objectives. Educators who wish to learn more about how the TEAMS objectives are tested should consult the TEAMS objectives and measurement specifications available at cost from TEA. The level of detail provided in this instructional strategies guide should provide sufficient information for educators to design effective instructional sequences. Second, each instructional strategies guide provides suggestions for instructional activities that can be used to promote mastery of TEAMS objectives.

The specific setting in which a given district chooses to provide instruction in the TEAMS objectives will influence the way in which the materials in this guide can be employed. The format of the guide permits flexible use.

General Nature of the Test Items

The TEAMS Grade 7 mathematics objectives are measured by multiple-choice items that present a problem followed by four answer choices, one of which is correct. The items, depending on the objective, present a graphic or tabular display, a geometric figure, a numeric problem, or a word problem. The problems are similar to those that students encounter in school, at home, or in their community.

SKILL AREA: NUMBER CONCEPTS

The skill area of number concepts consists of one objective involving conversion of decimals and percents to equivalent forms.

Objective 1: Convert decimals or percents to equivalent forms.

This objective requires students to convert decimals or percents to equivalent forms.

Sample Item

Change 6% to a fraction.

A. $\frac{6}{10}$

B. $\frac{1}{6}$

C. $\frac{10}{6}$

D. $\frac{6}{100}$

Test Questions

1. A test item will require students to carry out one of four possible conversions: (1) a decimal to an equivalent fraction, (2) a decimal to an equivalent percent, (3) a percent to an equivalent decimal, or (4) a percent to an equivalent fraction.
2. Percents will be expressed using the symbol % and will be whole numbers less than 100.
3. Decimals will have a value less than one and will be limited to tenths or hundredths.

Answer Choices

1. Each answer choice for a given test item will be either a fraction, a decimal, or a percent.
2. The correct answer choice will be the fraction, decimal, or percent that is equivalent to the specified number.
3. Answer choices will not include fractions that are equivalent to each other.

4. Fractional answer choices will not be reduced.
5. Decimal answer choices will be limited to tenths or hundredths.
6. Incorrect answer choices are likely to result from one or more of the following types of errors:
 - a. Computation Error: An error in the addition, subtraction, multiplication, or division of two numbers.
 - b. Misplaced Decimal Point: Placing the decimal point in an incorrect position on either a correct solution or an incorrect solution. A misplaced decimal point is the only type of error that will be used for incorrect decimal answer choices.
 - c. Reciprocal Error: Creating a number that is the reciprocal of the correct answer. For example, if the correct answer is $\frac{2}{3}$, identifying $\frac{3}{2}$ as the correct answer would be a reciprocal error.
 - d. Place Value Error: Failing to retain the place value of zero.
 - e. Fraction Conversion Error: An error in converting a decimal or percent to a fraction.

Answer Analysis for Sample Item

1. Answer choice A is incorrect because of a place value error. A zero was dropped from the denominator. The resulting fraction, $\frac{6}{10}$, is equivalent to 60% not 6%.
2. Answer choice B is incorrect because of a fractional conversion error. In converting to a fraction, 1 was placed over 6, instead of 6 over 100.
3. Answer choice C is also incorrect because of a fractional conversion error. Rather than placing 6 in the numerator, 6 was placed in the denominator.
4. Answer choice D is the correct response. The fraction equivalent of 6% is $\frac{6}{100}$ because percent expresses a number as a proportion of 100.

Instructional Emphases

After stressing the importance of knowing which conversion is called for, you may want to review the following procedures for each type of conversion.

Converting Decimals to Fractions

1. Note the place value of the last digit in the decimal.
2. Drop the decimal point and place the entire number over the above-noted place value.

Example: .25 = what fraction?

Step 1: .25 = hundredths

Step 2: $\frac{25}{100}$

Converting Decimals to Percents

1. Multiply by 100 (move the decimal point two places to the right).
2. Add a percent sign.

Example: 0.81 = what percent?

Step 1: 81.

Step 2: 81%

Converting Percents to Decimals

1. Drop the percent sign.
2. Divide by 100 (move the decimal point two places to the left).

Example: 20% = what decimal?

Step 1: 20

Step 2: .20

Converting Percents to Fractions

1. Drop the percent sign.
2. Place the number over 100.

Example: 10% = what fraction?

Step 1: 10

Step 2: $\frac{10}{100}$

To master the objective of equivalencies students must be able to:

1. Recognize the form of fractions $\frac{A}{B}$,
decimals .X or .XX, and percents X%.
2. Multiply and divide by 100.

Be sure to provide instruction in these areas if students are having difficulty. It may also be helpful for students to memorize a few common conversions such as $.25 = \frac{25}{100} = 25\%$.

Students can then refer to these memorized relationships to determine the method necessary to perform other, less common, conversions.

Possible Instructional Activities

Almanac Fact Conversions

Have students convert these numerical facts taken from the almanac to a fraction, decimal, or percentage, as specified.

1. In the U.S. about 26% of the population lives in rural areas.
fraction = _____.
2. In 1985, 40% of the mayors from the ten largest U.S. cities were black.
fraction = _____.
3. About .21 of the population of Texas live in the Dallas/Fort Worth metropolitan area.
percentage = _____.
4. At least 52% of the state and local government employees in the U.S. work in education.
fraction = _____.
5. The literacy rate in China is now estimated to be 75%.
fraction = _____.
6. The main religion in Britain is Anglicanism, to which 66% of the population belong.
decimal = _____.
7. About 78% of the earth's atmosphere is composed of nitrogen.
fraction = _____.

What's on a Label?

Have students select a food product that has a nutrition label listing the percentage of nutrients per serving (such as protein, vitamin A, vitamin C) as illustrated below.

Raisin Muffins

Percentage of U.S Recommended Daily Allowance (U.S. RDA) Per Serving

Protein	8%	Riboflavin	10%
Vitamin A	*	Niacin	10%
Vitamin C	*	Calcium	4%
Thiamine	15%	Iron	10%

* Contains less than 2% of the U.S. RDA
of these nutrients.

Have students convert the given percentages to their equivalent decimals and fractions. Students should disregard those nutrients for which no specific RDA is supplied.

Class Survey

Conduct a class survey and determine the proportion of people in the class (expressed as a decimal number) who fall into categories such as the following:

- own a dog
- play the guitar
- speak a foreign language
- are left-handed
- are sleepwalkers

Present the decimal numbers to the class. Call on different students to come to the chalkboard to convert each decimal number to their equivalent percentages and fractions.

SKILL AREA: COMPUTATION

The skill area of computation contains two objectives, namely, computation with fractions and computation with decimals.

Objective 2: Add and subtract fractions with like or unlike denominators.

This objective requires students to add and subtract fractions with like or unlike denominators.

Sample Item

Solve the problem. Express your answer in lowest terms.

$$\frac{5}{7} + \frac{3}{14} = \underline{\hspace{2cm}}$$

- A. $\frac{4}{7}$
- B. $\frac{6}{7}$
- C. $\frac{8}{21}$
- D. $\frac{13}{14}$

Test Questions

1. A test item will present either an addition or subtraction problem involving fractions. Students will be directed to express their answer in lowest terms.
2. Some items will involve unlike denominators. Least common denominators will not exceed 16.
3. A fraction presented in any addition problem will have a value less than one.
4. Both horizontal and vertical problem formats may be used.
5. In addition problems, the sum of the two addends will be less than one.

Answer Choices

1. All answer choices will be fractions that are expressed in lowest terms.
2. The correct answer will be the exact solution to the addition or subtraction problem presented.
3. Incorrect answer choices are likely to result from one or more of the following types of errors:
 - a. Computation Error: An error in the addition or subtraction of two numbers.
 - b. Common Denominator Error: An error in selecting the lowest common denominator or in renaming fractions in terms of the lowest common denominator.
 - c. Incorrect Operation: Performing an operation other than the required one.

Answer Analysis for Sample Item

1. Answer choice A is incorrect because of a common denominator error. Although the correct lowest common denominator (LCD), 14, was used, the numerator of the second fraction was not converted to match the denominator. Thus, the two numerators were simply added ($5 + 3$) and placed over the common denominator to obtain $\frac{8}{14}$. This fraction was then correctly reduced to $\frac{4}{7}$.
2. Answer choice B is incorrect because of a computation error. Although $\frac{5}{7}$ was correctly converted to $\frac{10}{14}$, an error was made in adding the numerators of the two fractions ($10 + 3$ is not 12). The incorrect answer, $\frac{12}{14}$, was then simplified to $\frac{6}{7}$.
3. Answer choice C is incorrect because of an incorrect operation. The sum of the numerators ($5 + 3$) was simply placed over the sum of the denominators ($7 + 14$) to obtain $\frac{8}{21}$.
4. Answer choice D is the correct response, as illustrated below.

Step 1: (change first fraction to match LCD) $\frac{10}{14}$

Step 2: (add first and second fractions) $\frac{10}{14} + \frac{3}{14} = \frac{13}{14}$

Step 3: (simplify fraction, if possible) $\frac{13}{14}$ cannot be simplified

Instructional Emphases

The problems testing this objective require students to perform addition and subtraction with fractions and express their answers in simplest terms. Emphasize to students that when adding or subtracting two fractions the denominators are neither added nor subtracted. Only the two numerators are added or subtracted to arrive at the answer. In problems where the denominators are not alike, students must know how to determine a common denominator and rename the fractions accordingly before performing the specified operation.

The steps needed to solve problems that involve addition and subtraction of fractions are presented below. You may want to review these steps with students as you work through several sample problems. In particular it may be useful to stress the steps used to find the LCD.

Addition and Subtraction of Fractions

1. Check the denominators of the fractions. Are they the same? If so, proceed to Step 4.

2. If the denominators are not the same, find a common denominator, preferably the LCD. To determine the LCD of two (or more) fractions, list the multiples of the denominators until a multiple common to both numbers is found. That multiple is the LCD of the two fractions. For example, to find the LCD of $\frac{1}{6}$ and $\frac{3}{8}$, list the multiples of 6 and 8 until a common multiple is found.

Multiples of 8: 8, 16, 24
Multiples of 6: 6, 12, 18, 24

Thus, 24 is the LCD.

3. Rewrite the fractions with the LCD as the new denominators. That is, for each fraction, multiply the numerator by the same number that was used to convert the original denominator into the LCD. You may want to remind students that multiplying both the numerator and denominator of a fraction by the same number is equivalent to multiplying that fraction by 1. Any number multiplied by 1 is the number itself. Therefore, multiplying both the denominator and numerator by the same number results in an equivalent fraction. Thus:

$$\frac{1}{6} = \frac{1 \times 4}{6 \times 4} = \frac{4}{24}, \text{ and}$$

$$\frac{3}{8} = \frac{3 \times 3}{8 \times 3} = \frac{9}{24}.$$

4. Perform the specified operation (addition or subtraction) on the numerators only. Use the same denominator in the answer.

$$\frac{4}{24} + \frac{9}{24} = \frac{13}{24}$$

5. Simplify the resulting fraction, if possible, by dividing the numerator and denominator by the greatest common factor (that is, the largest number that divides evenly into both numbers).

Example A: $\frac{1}{8} + \frac{3}{8}$

Step 1: denominators are the same

Step 4: $\frac{1}{8} + \frac{3}{8} = \frac{4}{8}$

Step 5: $\frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$

Example B:

Step 1: denominators are not the same

Step 2: LCD = 8

Step 3: $\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$

Step 4: $\frac{6}{8} - \frac{3}{8} = \frac{3}{8}$

Step 5: $\frac{3}{8}$ cannot be simplified

Possible Instructional Activities

Fraction Four-Square

Draw these diagrams on the board. Place fractions and operation signs as shown below.

DIAGRAM A

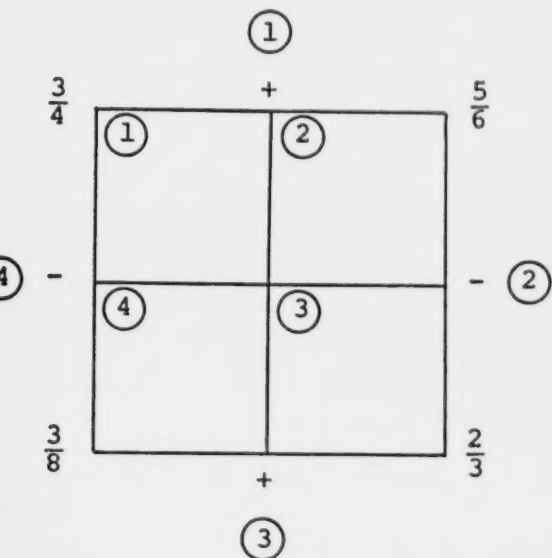
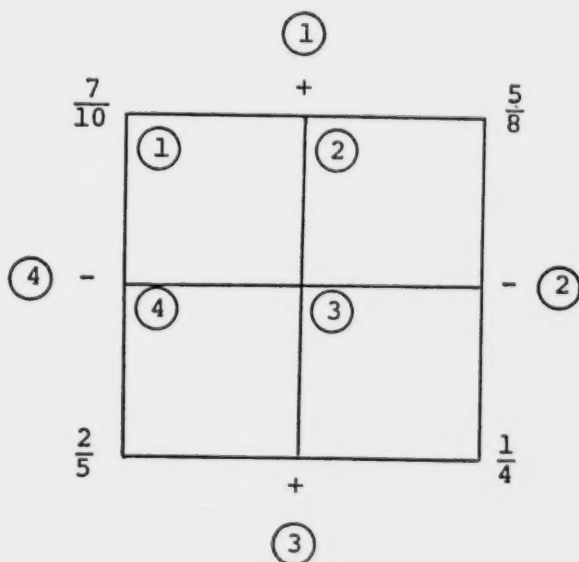


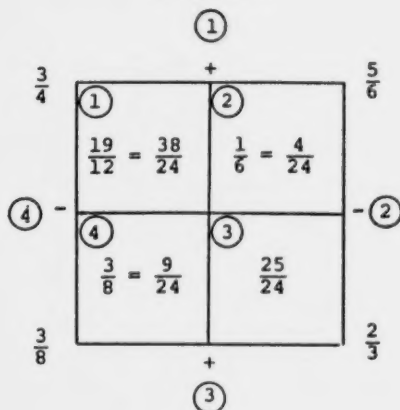
DIAGRAM B



For each diagram, four operations are specified. Tell students to perform each specified operation on a piece of paper, starting at the upper left corner and moving clockwise. Then call on students to supply each answer. Write the answer to each of the four operations in the appropriate boxes. For each subtraction operation, make certain that students subtract the smaller, bottom fraction from the larger, top fraction as illustrated in the solution shown below. As shown, the answers are simplified then written in their appropriate boxes.

For each completed diagram, ask students to double-check their answer by adding the fractions in boxes 2, 3, and 4. The sum should be equal to the fraction in box 1 if all operations were performed correctly.

Diagram A

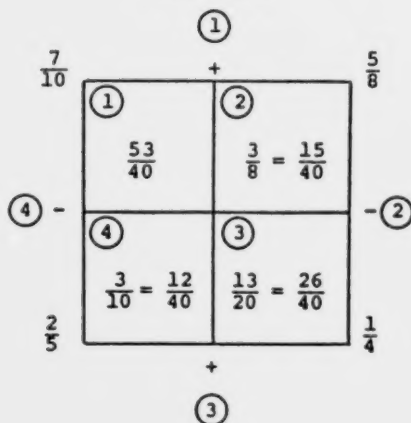


1. $\frac{3}{4} + \frac{5}{6} = \frac{9}{12} + \frac{10}{12} = \frac{19}{12}$
2. $\frac{5}{6} - \frac{2}{3} = \frac{5}{6} - \frac{4}{6} = \frac{1}{6}$
3. $\frac{2}{3} + \frac{3}{8} = \frac{16}{24} + \frac{9}{24} = \frac{25}{24}$
4. $\frac{3}{4} - \frac{3}{8} = \frac{6}{8} - \frac{3}{8} = \frac{3}{8}$

LCD = 24

Check: $\frac{4}{24} + \frac{25}{24} + \frac{9}{24} = \frac{38}{24}$ same as box 1

Diagram B



1. $\frac{7}{10} + \frac{5}{8} = \frac{28}{40} + \frac{25}{40} = \frac{53}{40}$
2. $\frac{5}{8} - \frac{1}{4} = \frac{5}{8} - \frac{2}{8} = \frac{3}{8}$
3. $\frac{1}{4} + \frac{2}{5} = \frac{5}{20} + \frac{8}{20} = \frac{13}{20}$
4. $\frac{7}{10} - \frac{2}{5} = \frac{7}{10} - \frac{4}{10} = \frac{3}{10}$

LCD = 40

Check: $\frac{15}{40} + \frac{26}{40} + \frac{12}{40} = \frac{53}{40}$ same as box 1Find the Factor

Provide students with several series of equivalent fractions. The equivalent fractions in each series should be ordered by consecutive increasing or decreasing multiples of the fully reduced fraction. Omit any one fraction other than the fully reduced fractions from the series and leave a blank space in the list. Next to each set write four possible factors, one of which is the factor that will express the missing fraction in its simplest terms. Below are two examples of such problems.

1. $(\frac{20}{45}, \frac{16}{36}, \text{---}, \frac{8}{18}, \frac{4}{9})$ 2 3 5 6

2. $(\frac{3}{4}, \frac{6}{8}, \text{---}, \frac{12}{16}, \frac{15}{20})$ 3 4 6 8

Have students fill in the missing fraction and then circle the factor corresponding to the missing fraction.

Opposite Pairs

Provide students with sets of fractions presented in pairs. Have students perform both addition and subtraction with the fractions in each pair. Always place the larger fraction on the left so that subtraction will result in a positive value. For example, given $\frac{7}{16}$ and $\frac{1}{8}$, students would perform the following operations:

$$\frac{7}{16} + \frac{1}{8} = \underline{\hspace{2cm}} \quad \text{and} \quad \frac{7}{16} - \frac{1}{8} = \underline{\hspace{2cm}}$$

Tell students to express their answers in lowest terms. Go over the correct answers in class.

Objective 3: Add, subtract, and multiply decimals.

This objective requires students to solve addition, subtraction, and multiplication problems involving decimal numbers.

Sample Item

$$46 - 16.09 = \underline{\hspace{2cm}}$$

- A. 29.91
- B. 30.91
- C. 30.09
- D. 29.1

Test Questions

1. Numbers in addition and subtraction items will not be larger than five digits. For each item, the numbers to be added or subtracted will have different numbers of places (up to three) to the right of the decimal point.
2. In multiplication items, factors may not contain more than three digits. Any three-digit factor may only be multiplied by a one-or two-digit factor.
3. Addition and subtraction items will be presented horizontally. Multiplication items will be presented either horizontally or vertically.
4. Addition items will have no more than three addends. Multiplication items will have no more than two factors.

Answer Choices

1. The correct answer choice will be the exact solution to the addition, subtraction, or multiplication problem presented.
2. Incorrect answer choices are likely to result from one or more of the following types of errors:
 - a. Computation Error: An error in the addition, subtraction, or multiplication of two numbers.
 - b. Regrouping Error: Failing to regroup when necessary, regrouping incorrectly, or regrouping unnecessarily.

- c. Misplaced Decimal Point: Placing the decimal point in an incorrect position on either a correct solution or an incorrect solution.
- d. Misplaced Digit: Either (1) adding or subtracting a digit of one place value to or from a digit of an adjacent place value or (2) misaligning partial products.
- e. Place Value Error: Failing to retain the place value of zero.

Answer Analysis for Sample Item

1. Answer choice A is the correct response, as illustrated. The subtraction is accurate, the digits are correctly aligned, and the decimal point is properly placed.

$$\begin{array}{r} 46.00 \\ - 16.09 \\ \hline 29.91 \end{array}$$
2. Answer choice B is incorrect because of a regrouping error, as illustrated. There was a failure to regroup after borrowing from the ones place.

$$\begin{array}{r} 46.00 \\ - 16.09 \\ \hline 30.91 \end{array}$$
3. Answer choice C is incorrect because of a computation error, as illustrated. The numbers in the hundredths place were subtracted incorrectly. The value of the subtrahend (.09) was used in the solution.

$$\begin{array}{r} 46.00 \\ - 16.09 \\ \hline 30.09 \end{array}$$
4. Answer choice D is incorrect because of a failure to retain the place value of zero in the subtrahend, as illustrated. Because the zero in the subtrahend (16.09) was omitted, the 9 became misaligned in the subtraction problem.

$$\begin{array}{r} 46.0 \\ - 16.9 \\ \hline 29.1 \end{array}$$

Instructional Emphases

Discuss the similarities between operating on decimal numbers and operating on whole numbers. Remind students that whole numbers are equivalent to decimal numbers with zeros to the right of the decimal point. Presented below are the steps required to solve problems involving addition, subtraction, and multiplication with decimal numbers. It may be helpful to review these steps with students as you present sample problems.

Adding and Subtracting with Decimals

1. Write the problem in vertical format. Align the decimal points of the numbers in the problem. This will ensure that the place-value positions are aligned correctly.
2. To avoid alignment errors, add 0's so that all numbers in a problem have the same number of digits to the right of the decimal point.
3. Perform computations. Align the decimal point in the answer with the decimal points in the problem.

Example A: $5.6 + 4 + 3.21$

$$\begin{array}{r} \text{Step 1:} \quad 5.6 \\ \quad \quad 4 \\ \quad \quad + 3.21 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Steps 2 and 3:} \quad 5.60 \\ \quad \quad 4.00 \\ \quad \quad + 3.21 \\ \hline \quad \quad 12.81 \end{array}$$

Example B: $12.2 - 9.11$

$$\begin{array}{r} \text{Step 1:} \quad 12.2 \\ \quad \quad - 9.11 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Steps 2 and 3:} \quad 12.20 \\ \quad \quad - 9.11 \\ \hline \quad \quad 3.09 \end{array}$$

Multiplying with Decimals

1. Write the problem in vertical format. Align digits on the far right, as in whole number operations.
2. Multiply, as with whole numbers.
3. Place the decimal point in the answer such that the number of places to the right of the decimal is equal to the sum of the number of places to the right of the decimals in the numbers being multiplied.

Example: $6.26 \times .6$

$$\begin{array}{r} \text{Step 1:} \quad 6.26 \\ \quad \quad \times .6 \\ \hline \end{array}$$

Step 2:
$$\begin{array}{r} 6.26 \quad (2 \text{ places to right of decimal}) \\ \times .6 \quad (1 \text{ place to right of decimal}) \\ \hline \end{array}$$

Step 3: $3.\overline{756}$ There are a total of $2 + 1$, or 3 places to the right of the decimal in the factors.

Possible Instructional Activities

Point Blank

Have students complete these problems by inserting the decimal point in the answer.

$$\begin{array}{r} 45.6 \\ \times 3.4 \\ \hline 15504 \end{array}$$

$$\begin{array}{r} 69.4 \\ \times 16 \\ \hline 11104 \end{array}$$

$$\begin{array}{r} .829 \\ \times 3 \\ \hline 2487 \end{array}$$

$$\begin{array}{r} 1.57 \\ \times 3.8 \\ \hline 5966 \end{array}$$

$$17.9 + .6 = 185$$

$$4.68 + 957 = 96168$$

$$143.9 + 8 = 1519$$

$$3.85 + 1.9 = 575$$

$$9.74 - 1.8 = 794$$

$$45 - 23.47 = 2153$$

$$168.7 - 99 = 697$$

$$175.08 - 86.2 = 8888$$

Palindromes

Tell students that numbers which can be read the same forward and backward are called "palindromes." The answer to each of these exercises is a palindrome.

a. $66.15 - 15 =$

d. $233.2 \times .2 =$

b. $33.3 \times 1.1 =$

e. $40.06 + 28.8 =$

c. $10.2 + 5 + .31 =$

f. $36.31 - 24.1 =$

Digit Pathway

Give students the following "pathway" problem. Tell students to fill in the blanks by solving each operation in the pathway, moving clockwise from the starting point. As shown, the answer to each operation is used as an operand in the subsequent operation. You may want to tell students that if the series of computations are performed correctly, the last box should contain a 4-digit number with the same numeral repeated in each place value.

Start

$$21.27 + 302.3 = \boxed{} - 300.67 = \boxed{}$$

$$\begin{array}{r} \times 1.4 \\ \hline \end{array}$$

$$\boxed{}$$

$$\begin{array}{r} - 25.4 \\ \hline \end{array}$$

$$\boxed{}$$

$$\begin{array}{r} \times 9 \\ \hline \end{array}$$

$$\boxed{}$$

$$\begin{array}{r} - 48.83 \\ \hline \end{array}$$

$$\boxed{}$$



SKILL: PROBLEM SOLVING

The skill area of problem solving contains two objectives, namely, solving two-step word problems with whole numbers and solving two-step word problems with decimals.

Objective 4: Solve two-step word problems using the basic operations with whole numbers.

This objective presents word problems that require students to perform two computations involving the addition, subtraction, multiplication, or division of whole numbers.

Sample Item

Ellen needs 256 bricks to make a garden wall 8 feet high. She bought 260 bricks, but she broke 18 of them. How many more bricks does she need to finish the wall?

- A. 22 bricks
- B. 14 bricks
- C. 4 bricks
- D. 18 bricks

Test Questions

1. A test item will present a word problem whose solution will require two computations. The operations tested will be addition, subtraction, multiplication, and division.
2. Problems will contain whole numbers of less than four digits.
3. Some test items will contain numerical information that is irrelevant to the solution of the problem.
4. Division problems will not yield remainders.
5. No test item will involve money.
6. The numbers used in computation will be written in numerical form.

Answer Choices

1. All answer choices will be appropriately labeled whole numbers.
2. The correct answer choice will be the exact solution to the problem presented.
3. Incorrect answer choices are likely to result from one or more of the following types of errors:
 - a. Computation Error: An error in the addition, subtraction, multiplication, or division of two numbers.
 - b. Regrouping Error: Failing to regroup when necessary, regrouping incorrectly, or regrouping unnecessarily.
 - c. Missing Operation: Not operating on the data or omitting one or more of the required operations.
 - d. Incorrect Operation: Performing an operation other than the required one.
 - e. Incorrect Data Error: Performing a necessary operation on the wrong data.
 - f. Incorrect Order Error: Performing the operations required to solve the problem but in the wrong order.

Answer Choice Analysis for Sample Item

1. Answer choice A is incorrect as the result of using the wrong data. In this answer choice the number of broken bricks is subtracted from the total needed, rather than from the total purchased. This difference is then subtracted from the number of bricks originally purchased.

$$\begin{array}{r} \text{Step 1: (error)} \quad 256 \\ \quad \quad \quad - 18 \\ \hline \quad \quad \quad 238 \end{array}$$

$$\begin{array}{r} \text{Step 2: (error)} \quad 260 \\ \quad \quad \quad - 238 \\ \hline \quad \quad \quad 22 \end{array}$$

2. Answer choice B is the correct response. All the calculations are set up correctly and performed accurately, as illustrated.

$$\begin{array}{r} \text{Step 1: } 260 \\ - 18 \\ \hline 242 \end{array}$$

$$\begin{array}{r} \text{Step 2: } 256 \\ - 242 \\ \hline 14 \end{array}$$

3. Answer choice C is incorrect because of a regrouping error in the second subtraction operation.
4. Answer choice D is incorrect because of missing operations. The number of broken bricks was selected as the solution to the problem without performing any computations.

Instructional Emphases

The key skills for this objective include deciding what data are necessary to solve the problem, determining what operations are needed, ordering those operations correctly, and performing the calculations accurately. At this grade level test items may contain irrelevant information; thus, it is essential that students be able to determine what data are necessary and what are not. You may want to review this step-by-step procedure with students:

1. Read the problem carefully.

Read the problem to determine its content. Pay particular attention to the question being asked.

2. Find the essential information.

Consider each piece of numerical information and decide whether it is needed to answer the question.

3. Determine the computations to be performed and their order.

Decide what computations will be needed to solve the problem. Be sure that the operations selected follow from the question asked.

4. Perform the necessary computations.

Check the completed computation against the problem to be certain that the right numbers and the correct operations were used. Then check the calculations.

5. Is the answer reasonable?

Reread the problem to make certain that the answer seems reasonable, given the situation presented.

The sample item can be used to illustrate these five steps:

Step 1: The problem is about building a brick wall. The question is about the number of bricks needed to complete the wall.

Step 2: The important information includes:

- a. 256 bricks are needed to build the wall.
- b. 260 bricks were bought.
- c. 18 bricks were broken.

The information that the wall is 8 feet high is irrelevant.

Step 3: To determine how many bricks are needed, subtract the number of broken bricks from the number of bricks that were bought. Then subtract this difference from the number of bricks needed to build the wall, as shown:

$$256 - (260 - 18) = X$$

Step 4: The answer calculated is 14 bricks.

Step 5: The answer appears reasonable, given the numbers involved.

Possible Instructional Activities

Detective Story

Give students brief descriptions of problems. Ask them what information is needed in order to solve each problem. Provide a list of possible pieces of information, including some information that is irrelevant. Let students select the needed information then discuss their choices. You can use word problems from workbooks or texts to supply the situations. The following is an example:

Felipe is building a bookcase. He plans to use 5 boards and 35 nails. He wants to know how much the bookcase will weigh when it is finished. Which of these facts does he need to know?

- a. the length of the bookcase
- b. the weight of a board
- c. the weight of the books
- d. the cost of the materials
- e. the weight of a nail

The correct answers are b and e.

In These Times

Have students use newspaper articles to create word problems. Tell students to base their problems on articles about current events, sports, business, weather, or entertainment. Emphasize that students should feel free to change the story or numbers to make a suitable word problem. Ask students to include one piece of irrelevant information in the problem. Collect the problems and present some of them to the class to solve.

Say It with Numbers

Design a matching drill to help students recognize the order in which operations must be carried out. In one column, create word problems that require various operations and orders of computation. Use the same numbers in every problem. One of these numbers should describe irrelevant information.

In a second column, make a list of equations. Tell students to match each problem to the appropriate equation, as in the example below. In addition to the correct equations, you may also include equations which will not solve any of the word problems.

- | | |
|--|---------------------------|
| 1. Members of the Gonzalez family grew tomatoes in their garden for 10 weeks. They harvested 84 tomatoes from their garden. They gave 6 tomatoes to each of their 7 neighbors. How many tomatoes did the Gonzalez family have left to eat? (a) | a. $84 - (6 \times 7)$ |
| | b. $(84 \div 7) \times 6$ |
| | c. $(84 + 10) \times 6$ |
| | d. $84 - 6 - 7$ |
| | e. $84 + 7 - 10$ |
| 2. Arnold started his stamp collection with 7 foreign stamps. During the next year he collected 84 more, but lost 10. A total of 6 of his stamps are from Australia. How many stamps does Arnold have? (e) | f. $(84 - 10) \div 7$ |

3. Christine sold 84 pairs of shoes over the weekend. On Monday, 6 of the 10 most expensive pairs were returned, and on Tuesday, 7 more pairs were returned. How many pairs were not returned? (d)
4. Howard loaned a total of 84 books to 7 people. Each person got the same number of books. After 10 weeks, 6 people had returned Howard's books. How many books did Howard get back? (b)

Objective 5: Solve two-step word problems using addition, subtraction, and multiplication with decimals.

This objective requires students to solve word problems by performing two computations involving the addition, subtraction, or multiplication of decimal numbers.

Sample Item

Jim is shopping for clothes. He finds some sweaters that cost \$17.50 each and shirts that cost \$12.95 each. If he buys 1 sweater and 3 shirts, how much will he spend? (Assume no sales tax.)

- A. \$30.45
- B. \$91.35
- C. \$54.35
- D. \$56.35

Test Questions

1. A test item will present a word problem that requires students to perform two computations using decimal numbers. The operations tested will be addition, subtraction, and multiplication.
2. The numbers presented in items will contain a maximum of four digits, with no more than three digits to the right of the decimal.
3. For multiplication problems, the total number of digits will not exceed five (that is, a three-digit number multiplied by a two-digit number or a four-digit number multiplied by a one-digit number).
4. No test item will contain numerical information that is irrelevant to the solution of the problem.
5. At least one problem will involve money.

Answer Choices

1. All answer choices will be appropriately labeled.
2. The correct answer choice will be an exact solution to the problem presented.

3. Incorrect answer choices are likely to result from one or more of the following types of errors:
 - a. Computation Error: An error in the addition, subtraction, multiplication or division of two numbers.
 - b. Regrouping Error: Failing to regroup when necessary, regrouping incorrectly, or regrouping unnecessarily.
 - c. Missing Operation: Not operating on the data or omitting one of the required operations.
 - d. Misplaced Decimal Point: Placing the decimal point in an incorrect position on either a correct solution or an incorrect solution.
 - e. Incorrect Operation: Performing an operation other than a required one.
 - f. Incorrect Data Error: Performing a necessary operation on the wrong data.
 - g. Incorrect Order: Performing the operations required to solve the problem, but in the wrong order.
 - h. Misaligned Digit: Either (1) adding or subtracting a digit of one place value to or from a digit of an adjacent place value or (2) misaligning partial products.

Answer Analysis for Sample Item

1. Answer choice A is incorrect because of a missing operation. The price of the sweater and of the shirt are added (\$17.50 + \$12.95). Thus, the step in which the price of a shirt is multiplied by 3 is omitted.
2. Answer choice B is incorrect because of an incorrect order in performing the operations. The cost of the shirt and of the sweater are added together, then multiplied by 3.
3. Answer choice C is incorrect because of a regrouping error in the first step, as illustrated below:

Step 1:	(regrouping	\$12.95	
	error)	$\times \quad 3$	shirts
		\$36.85	

Step 2:	(incorporates	\$36.85
	error from	$+ 17.50$
	step 1)	
		\$54.35

4. Answer choice D is the correct response, as illustrated below:

$$\begin{array}{r} \text{Step 1: } \$12.95 \\ \times \quad 3 \text{ shirts} \\ \hline \$38.85 \end{array}$$

$$\begin{array}{r} \text{Step 2: } \$38.85 \text{ price of shirts} \\ + \$17.50 \text{ price of sweater} \\ \hline \$56.35 \end{array}$$

Instructional Emphases

For this objective, students must be able to solve two-step word problems and compute with decimal numbers. Thus, you may want to review problem-solving and computation with decimals separately. When you review word problems, you could use the step-by-step procedure described for Objective 4. When you review computation with decimals, be sure to reread the instructional emphases for Objective 3 (add, subtract, and multiply decimals). You may want to discuss the fact that the basic procedures for solving word problems are not affected by the types of numbers (whole vs. decimal) included in the problems. If students have difficulty with problems measuring this objective, try to determine whether the errors they make result from performing inappropriate operations or from computing inaccurately. Provide additional instruction and practice specific to the area of difficulty.

Possible Instructional Activities

Moving In

Carmelita is helping to build a playhouse in the backyard for her little brother.

1. To make some shelves, Carmelita needs 2 pieces of plywood that are each 2.6 feet long. She has a piece of plywood that is 6 feet long. How much plywood will be left after making the shelves?
_____ feet.
2. She needs to buy screwdrivers of different sizes that cost \$2.70 each and a hammer that costs \$7.95. If she buys 3 screwdrivers and 1 hammer, how much will she spend? (Assume no sales tax.) \$_____.
3. Carmelita also wants to buy some paint that costs \$4.86 per gallon. If she pays for 2 gallons of paint with a 20-dollar bill, how much change should she get back? \$_____.

Shopping Spree

Tell students to imagine that they are considering buying any two of the following items. For each pair of items they select, have students find the amount of change they would receive after paying for the items with a 50-dollar bill. (Assume no sales tax.)

cassette tape	\$8.00	record	\$ 6.95
pen	\$10.50	pencil	\$.39
shoes	\$26.48	pair of	\$ 2.25
sweater	\$19.00	shirt	\$12.50
book	\$ 7.35	magazine	\$ 1.95

Pizza Time

Distribute copies of the following pizza menu to students. Have students use the menu to make up problems for classmates to solve, or have students solve problems that you pose. Below are some examples. (Assume no sales tax.)

<u>Pizza</u>	<u>Small</u>	<u>Medium</u>	<u>Large</u>
Cheese	3.50	6.75	9.80
Pepperoni	4.00	7.50	10.30
Sausage	3.80	7.50	10.30
Canadian bacon	3.80	7.30	10.15
Mushroom	3.75	7.20	10.00

Add \$0.80 for each extra topping.

1. How much more would it cost to buy a large, cheese pizza than a medium sausage cheese pizza with 1 extra topping? \$_____.
2. Irene and Angelina ordered 2 medium Canadian bacon pizzas and 1 medium sausage pizza. How much was their bill? \$_____.
3. Paul bought 1 small mushroom pizza with 3 extra toppings. How much did he spend? \$_____.

SKILL AREA: MEASUREMENT

The skill area of measurement consists of one objective that involves conversion of measurement units.

Objective 6: Convert measurements within either the metric or customary system of measurement.

This objective requires students to convert a quantity from one unit of measure to another, using a conversion equation that is provided.

Sample Item

6 yards = _____ inches
(36 inches = 1 yard)

- A. 6 inches
- B. 186 inches
- C. 216 inches
- D. 72 inches

Test Questions

1. A test item will require students to convert a quantity from one unit of measure to another. Each test item will provide the relevant statement of equivalence, giving the smaller unit first (for example, 1000 meters = 1 kilometer).
2. No test item will require conversion between metric and customary units.
3. The properties of measurement that will be involved in conversion include weight/mass, length, and volume/capacity. The units of measure that are eligible for testing include:

METRIC

Length: millimeter, centimeter, meter, kilometer
Weight/Mass: milligram, gram, kilogram
Capacity/Volume: milliliter, liter, kiloliter

CUSTOMARY

Length: inch, foot, yard, mile
Weight/Mass: ounce, pound, ton
Capacity/Volume: cup, pint, quart, gallon

4. Quantities to be converted will be expressed as whole numbers or decimals.
5. Quantities expressed using the smaller unit in a statement of equivalence will not exceed four digits.

Answer Choices

1. The correct answer will be the exact solution to the problem presented.
2. Incorrect answer choices are likely to result from one or more of the following types of errors:
 - a. Computation Error: An error in the addition, subtraction, multiplication, or division of two numbers.
 - b. Regrouping Error: Failing to regroup when necessary, regrouping incorrectly, or regrouping unnecessarily.
 - c. Misplaced Decimal Point: Placing the decimal point in an incorrect position on either a correct solution or an incorrect solution.
 - d. Conversion Error: Using an incorrect conversion constant.
 - e. Incorrect Operation: Performing an operation other than a required one.

Answer Analysis for Sample Item

1. Answer choice A is incorrect because an incorrect operation is used. The conversion constant is used as a dividend rather than as a multiplication factor ($36 \div 6 = 6$).
2. Answer choice B is incorrect as the result of a regrouping error in the tens place, as illustrated below:

$$\begin{array}{r} 36 \\ \times 6 \\ \hline 186 \text{ inches} \end{array}$$

3. Answer choice C is the correct response. The correct operation is selected, and the computations are performed accurately.

$$\begin{array}{r} 3 \\ 36 \\ \times 6 \\ \hline 216 \text{ inches} \end{array}$$

4. Answer choice D is incorrect because of a conversion error. A conversion constant of 12, rather than 36, was used ($6 \times 12 = 72$).

Instructional Emphases

This objective involves performing one of two operations: (a) converting smaller units to larger units or (b) converting larger units to smaller units.

Students must be able to apply correctly a conversion constant in order to solve these types of problems. Explain to students that the conversion constant in a statement of equivalence is the number associated with the smaller units. For example, in the equation 2 pints = 1 quart, the 2 is the conversion constant.

Converting from smaller units to larger units is performed by dividing the number of smaller units by the conversion constant. Converting from larger units to smaller units is performed by multiplying the number of larger units by the conversion constant. Thus, when smaller units are converted to larger units, the conversion constant is a divisor; when larger units are converted to smaller units, the conversion constant is a multiplication factor.

You may wish to review the following procedures with your students:

Converting Smaller Units to Larger Units

Example: 64 ounces = _____ pounds

(16 ounces = 1 pound)

1. Study the problem. Note that the conversion required is from smaller units to larger units.
2. Divide the number in the problem labeled "ounces" by the conversion constant labeled "ounces":

$$\begin{array}{r} \text{(conversion} \quad 4 \text{ pounds} \\ \text{constant)} \quad 16 \overline{)64} \text{ ounces} \\ \underline{64} \\ 0 \end{array}$$

64 ounces = 4 pounds

Converting Larger Units to Smaller Units

Example: 5 pounds = _____ ounces

(16 ounces = 1 pound)

1. Study the problem. Note that the conversion required is from larger units to smaller units.
2. Multiply the number in the problem labeled "pounds" times the conversion constant labeled "ounces."

$$\begin{array}{r} 16 \text{ (conversion constant)} \\ \times 5 \text{ (pounds)} \\ \hline 80 \text{ ounces} \end{array}$$

5 pounds = 80 ounces

Possible Instructional Activities

Convert Away

Write the following conversion equations on the board one at a time. After each problem is written, read the equation aloud and ask students whether the problem requires multiplication or division to solve it.

17 millimeters	=	_____	centimeters
6 kilometers	=	_____	meters
26 pounds	=	_____	ounces
302 milligrams	=	_____	grams
2000 liters	=	_____	kiloliters
12 quarts	=	_____	cups
8000 pounds	=	_____	tons
36 feet	=	_____	yards
22 yards	=	_____	inches
41 gallons	=	_____	pints

After every problem has been discussed, tell students to solve the problems. Provide a conversion chart for students to use.

Measure Your Class

Divide the class into small groups. Give each group a list of 10 or so common classroom objects (for example, a math textbook, an eraser, a pen, etc.). Tell students to measure these objects, using both metric and customary units. Provide students with both metric and customary rulers to measure length. If available, students can also use a scale to measure weight and measuring cups to measure the capacities of various objects.

After taking the measurements, students should convert the measurements into other unit sizes. Supply students with both metric and customary conversion charts for this assignment.

Conversion Contest

Hold a conversion contest in class. Explain to students that you will write a quantity in specified units on the board. The students' task is to express that quantity in as many different units as possible within the same measurement system (that is, metric or customary). For example, 2000 milliliters, could be expressed as 2 liters or .002 kiloliters. Tell students they will have 5 minutes to perform the conversions. The student that performs the greatest number of accurate conversions in the time allotted is the winner.

You may or may not want to provide students with a conversion chart for this activity. Hold as many rounds of the contest as you think appropriate.

SKILL AREA: GEOMETRY

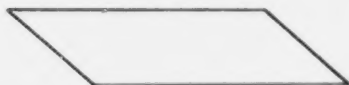
The skill area of geometry contains two objectives, namely, identify common geometric terms and figures and find the perimeter of polygons.

Objective 7: Identify common geometric terms and figures.

This objective requires students to link a geometric figure to the appropriate geometric term, or link a geometric term to the appropriate geometric figure.

Sample Item

The figure below is a picture of:



- A. a pentagon
- B. an octagon
- C. a parallelogram
- D. a diameter

Test Questions

1. A test item will either (a) present a figure and require students to select the matching geometric term, or (b) present a geometric term and require students to select the matching figure.
2. Geometric figures will meet the following specifications:
 - Lines will have arrows at both ends.
 - Trapezoids will be isosceles.
 - Parallelograms will not have right angles.
 - Polygons will be regular.
 - Circles will have center dots.

All of the following terms (and corresponding figures) will be eligible for testing.

a. Types of lines:

- intersecting lines
- perpendicular lines
- parallel lines

b. Parts of a circle:

- radius
- diameter
- circumference

c. Types of polygons:

- trapezoid
- parallelogram
- pentagon
- hexagon
- octagon

Answer Choices

1. The correct answer will be the term that corresponds to the figure presented or the figure that corresponds to the term presented.
2. All answer choices will be drawn from the list of terms eligible for testing.
3. When a test item requires students to select a term, all answer choices will be grammatically correct with respect to the question posed.
4. Incorrect answer choices will be a term or figure other than the one requested.

Answer Analysis for Sample Item

1. Answer choice A is incorrect because the figure depicted is not a pentagon.
2. Answer choice B is incorrect because the figure depicted is not an octagon.
3. Answer choice C is the correct response. The figure is correctly identified as a parallelogram.
4. Answer choice D is incorrect because the figure depicted is not a diameter.

Instructional Emphases

Identification of geometric figures requires that students are able to recognize the name, shape, and components of the figures. Moreover, students must be able to distinguish one type of figure from another. By seventh grade most students will be able to identify the more basic geometric figures and their names, such as circle and line. However, review of the different types of lines and the different components of a circle is appropriate. Also, emphasis should be placed on the polygonal figures with which most students have little experience. Exercises should help students discriminate between the various shapes and remember the appropriate names of shapes.

Students may benefit from learning about the sources of the names for the various geometric figures. English geometry terminology comes principally from Greek and Latin. The names for polygons come primarily from Greek, while many words relating to circles and lines come from Latin. Thus, poly is Greek for "many" and -gon is a Greek suffix meaning "angle." Specific polygon names are borrowed from the Greek system of numbers. Pente, the Greek word for "five," combined with the suffix -gon creates "pentagon" which means "five-angled."

The Greek names for numbers are listed below:

1	ena	
2	dya	
3	treis	
4	tettares	
5	pente	(as in pentagon)
6	hex	(as in hexagon)
7	hepta	(as in heptagon)
8	okto	(as in octagon)
9	ennez	
10	deka	(as in decagon)

Trapezoid comes from a Greek word trapezion, meaning "small table." Parallelogram also comes from Greek. You can use this information to create a lesson that will help students to understand how the terms were derived. This, in turn, may help them to remember the names.

Possible Instructional Activities

Geometric Draw

For this class activity, start by calling out a geometric term. Call on a student to draw the corresponding figure or component on the chalkboard. After the student has drawn the figure on the board, the rest of the class determines whether or not the figure is drawn correctly. Then, the student calls out

another geometric term and selects another student to draw that figure on the board. Repeat this process until several students have had a chance at the chalkboard.

Surrounded by Geometry

Have students search the classroom, the school, and their homes for examples of geometric figures. Some figures, such as circles, are easy to locate. Others will require some imagination. For example, patterns created by intersecting telephone lines create nearly all the geometric shapes. Tile floors can provide many good examples of geometric forms. Create a game by having students locate and record the shapes they see. Designate the shapes to be collected, then give students an opportunity to tell their classmates what they have found. As an extra challenge, have students measure the area or perimeter of the figures they find.

Artful Geometrics

Use templates to create designs made of polygons. Draw a pattern into the design such that if students locate and darken the proper figures, a message will be revealed. For example:

Shade in hexagons to find a popular dessert.



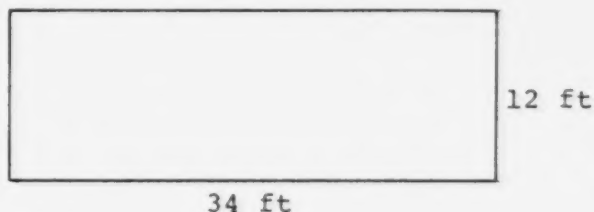
The answer is π , pi.

Objective 8: Find the perimeter of polygons.

This objective requires students to determine the perimeter of polygons.

Sample Item

What is the perimeter of the rectangle below?



- A. 408 ft
- B. 46 ft
- C. 92 ft
- D. 82 ft

Test Questions

1. A test item will consist of a polygon labeled with the minimum number of dimensions necessary to determine its perimeter.
2. The dimensions of a polygon will be given in whole numbers and labeled in appropriate units.
3. Regular polygons will be identified as such.
4. Formulas will not be provided.

Answer Choices

1. All answer choices will be whole numbers with appropriate labels.
2. The correct answer choice will be the exact solution to the problem presented.

3. Incorrect answers are likely to result from one or more of the following types of errors:
- Incorrect Operation: Performing an operation other than a required one.
 - Missing Operation: Not operating on the data or omitting one of the required operations.
 - Computation Error: An error in the addition, subtraction, multiplication, or division of two numbers.
 - Regrouping Error: Failing to regroup when necessary, regrouping incorrectly, or regrouping unnecessarily.
 - Incorrect Data Error: Performing a necessary operation on the wrong data.
 - Place Value Error: Failing to retain the place value of zero.

Answer Analysis for Sample Item

- Answer choice A is incorrect because the wrong operation is used on the data. Multiplying the length by the width ($34 \times 12 = 408$) results in the area, not the perimeter.
- Answer choice B is incorrect because of a missing operation. This answer is obtained by adding the two known dimensions ($34 + 12$). The resulting number, 46, must be multiplied by 2 to find the total length of all four sides.
- Answer choice C is the correct response. The correct formula was used, and computations are accurate as illustrated.

$$\begin{aligned} P &= 2(34) + 2(12) \\ &= 68 + 24 \\ &= 92 \end{aligned}$$

- Answer choice D is incorrect because of a regrouping error ($68 + 24 = 92$, not 82).

Instructional Emphases

To master this objective, students need to know the general geometric characteristics of different polygons. For example, they should know that parallel sides of a rectangle are of equal length and that all sides of a square are of equal length. In addition, they should be aware that the term regular specifies that all sides of a polygon are of equal length.

Explain to students that perimeter refers to the distance around a polygon. The perimeter can always be found by adding the lengths of the sides of a polygon. However, there are formulas to calculate the perimeter of particular shapes:

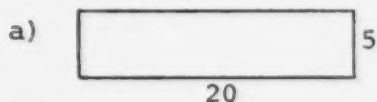
- a. rectangle: $P = 2 \text{ length} + 2 \text{ width}$
- b. regular polygon: $P = \text{length of a side} \times \text{number of sides}$
(including square)

Demonstrate how to determine perimeter using these formulas as well as the more general method of summing the lengths of the sides of a polygon. You may want to use the step-by-step procedures outlined below.

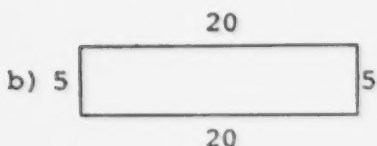
1. Read the question carefully. Determine whether the figure is a rectangle or another polygonal figure.

Rectangles

2. Copy the given dimensions onto the unlabeled, corresponding sides.



20



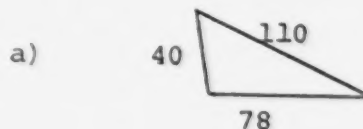
20

3. Add the four numbers together. The sum is the perimeter.

$$\begin{array}{r} 20 \\ 20 \\ 5 \\ + 5 \\ \hline 50 \end{array}$$

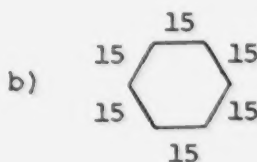
Other Polygons

2. If dimensions are given for all sides of the figure, add them together. The sum is the perimeter.



b)
$$\begin{array}{r} 110 \\ 78 \\ + 40 \\ \hline 228 \end{array}$$

3. If the dimensions are not all given, determine from the question if the polygon is regular; if it is, then all sides are equal in length. Write the given length on each side.



4. Alternatively, use the formula for the perimeter of a rectangle.

$$\begin{aligned} P &= 2 \text{ length} + 2 \text{ width} \\ &= 2 (20) + 2(5) \\ &= 50 \end{aligned}$$

4. Add together all the lengths in 3b; the sum is the perimeter.

$$\begin{array}{r} 15 \\ 15 \\ 15 \\ 15 \\ 15 \\ + 15 \\ \hline 90 \end{array}$$

5. Alternatively, multiply the length of the known side from 3a) by the total number of sides. The product is the perimeter.

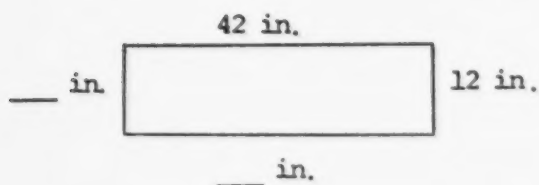
$$\begin{array}{r} 15 \\ \times 6 \\ \hline 90 \end{array}$$

Possible Instructional Activities

Perimeter of a 'Gon

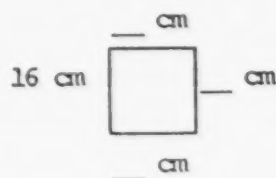
Distribute to students a sheet of problems similar to those below. Have students fill in the missing information for each figure by determining the lengths of the sides and the perimeter.

Rectangle



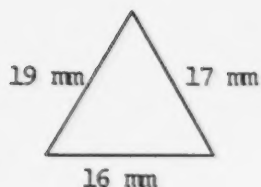
Perimeter = _____ in.

Square



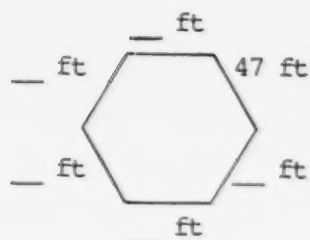
Perimeter = _____ cm

Triangle



Perimeter = _____ mm

Regular Polygon



Perimeter = _____ ft

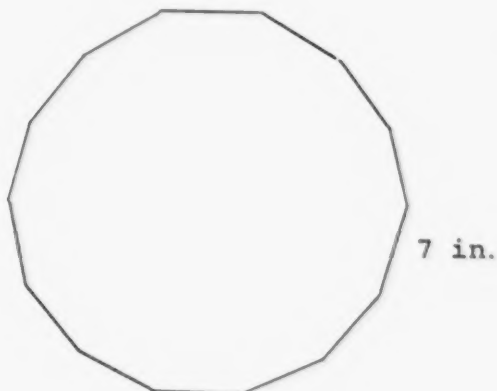
Supergon

You can use this exercise to help students to acquire the habit of numbering the sides of a polygon as they count them. Create regular polygons with a large number of sides. Give the length of one side. An example is shown below. Encourage students to carefully count the sides when they compute perimeter on polygons.

What is the perimeter of this regular polygon?

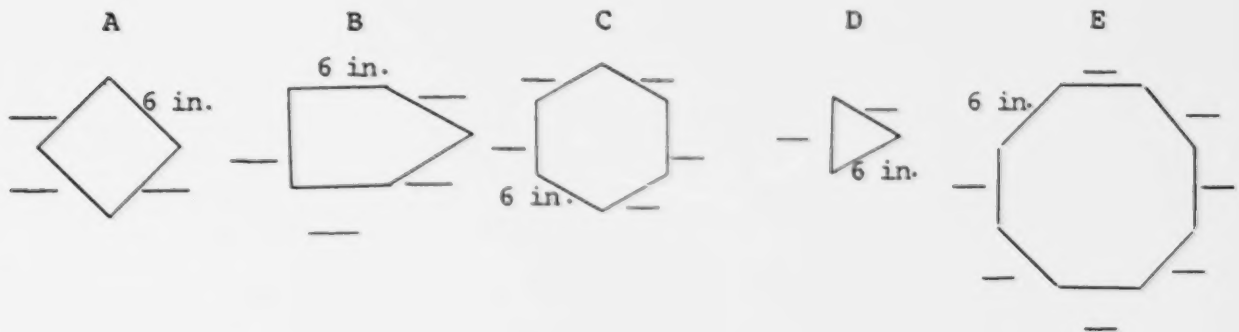
Solution:

$$\begin{array}{r} 14 \\ \times 7 \\ \hline 98 \end{array}$$



Lots of 'Gons

Present students with a series of similarly sized regular polygons. Within a series, each polygon should have a different number of sides but have the same length for each side. Have students count and label the sides and then compute the perimeter of each figure. An example of such a series is shown below.



Perimeter = _____

Classroom Perimeters

Have different groups of students measure and determine the perimeter of actual objects in the classroom. Specify objects such as the front cover of a book, a pencil (the hexagonal circumference), a desk top, and the chalkboard.

SKILL AREA: PROBABILITY, STATISTICS, AND GRAPHING

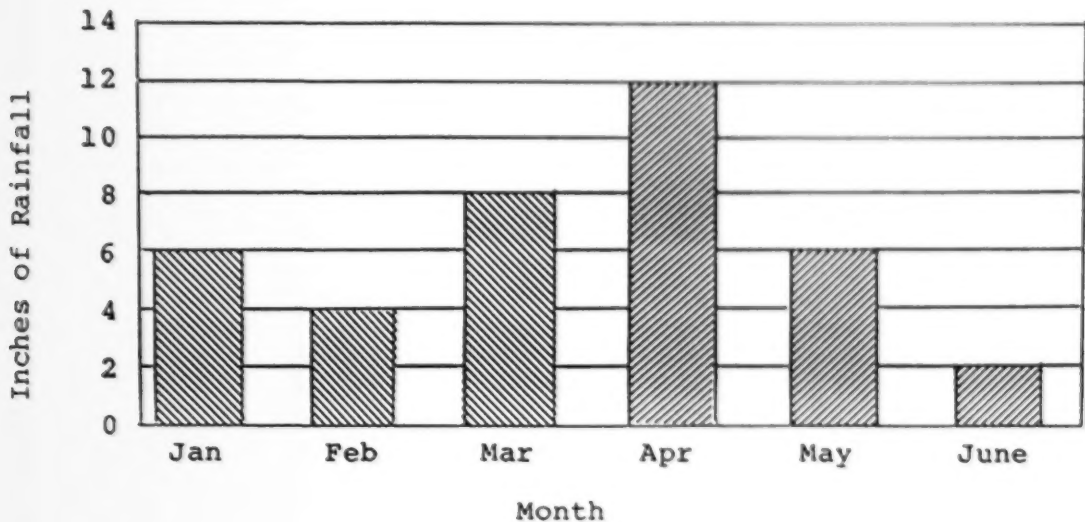
The skill area of probability, statistics, and graphing covers two objectives, namely, charts and graphs as well as possible outcomes.

Objective 9: Use information from charts or graphs to solve word problems.

This objective requires students to solve word problems using information from charts and graphs.

Sample Item

Rainfall for Houston, Texas



How many more inches of rain fell in April than in March?

- A. 8 inches
- B. 4 inches
- C. 6 inches
- D. 2 inches

Test Questions

1. A test item will present a graph or a table and will require that only one operation be applied to the data from the display.
2. The types of graphs that are eligible include bar, line, and picture graphs.
3. Values on numerically labeled axes of graphs will be expressed in whole numbers using intervals greater than one.
4. A graph will display no more than eight pieces of information. Students will not be required to interpolate data points between labeled intervals on bar and line graphs.
5. Bar graphs may include horizontal lines.
6. Line graphs will include a grid of both horizontal and vertical lines.
7. Line graphs will include large dots at data points.

Answer Choices

1. Each answer choice will be labeled appropriately.
2. The correct answer choice will be the exact solution to the problem presented.
3. Incorrect answer choices are likely to result from one or more of the following types of errors:
 - a. Computation Error: An error in the addition or subtraction of two numbers.
 - b. Incorrect Operation: Performing an operation other than the required one.
 - c. Regrouping Error: Failing to regroup when necessary, regrouping incorrectly, or regrouping unnecessarily.
 - d. Missing Operation: Not operating on the data or omitting one of the required operations.
 - e. Incorrect Selection of Information: Selecting the wrong information from a graph or chart.
 - f. Scale Error: Using a scale other than the one presented.

Answer Analysis for Sample Item

1. Answer choice A is incorrect because of a missing operation. The amount of rainfall in March (8 inches) was taken directly from the graph, and no operation was performed.
2. Answer choice B is the correct response. This response was obtained by subtracting the inches of rainfall in March from the inches of rainfall in April ($12 - 8 = 4$ inches).
3. Answer choice C is incorrect because of incorrect selection of information from the graph. Rather than using the data from March (8 inches), the data from May (6 inches) were used to solve the problem ($12 - 6 = 6$ inches).
4. Answer choice D is incorrect because of the use of a different scale. The scale interval is 2, not 1, as is implied by this response.

Instructional Emphases

This objective requires students to obtain information from a chart or graph and perform a computation to answer a related question. It may be a good idea to focus your instruction on how to read information from a chart or graph, as computation skills will be covered in a variety of other objectives.

To help students learn how to interpret a graph or chart, you may want to discuss various types of displays and their uses:

1. Bar Graphs:

A bar graph typically shows how different data compare, such as the height of various species of trees or the number of people who own different pets. The bars can be compared with each other according to their relative length and can be displayed either vertically or horizontally.

2. Line Graphs:

A line graph typically shows change in data, such as rainfall across various years or the number of student absences in various months.

3. Picture Graphs:

A picture graph, like a bar graph, shows how different data compare, such as the number of students in various schools. A symbol is used to represent a specified

number of items. For example, the symbol  could be used to represent 50 students. The number of students

per school could than be represented by the number of symbols displayed.

4. Charts:

A chart is used to organize and display (in rows and columns) large amounts of numeric information.

An important factor to consider when solving word problems for this objective is that graphs and charts contain both essential and nonessential information. Therefore, it is important for students to pinpoint what the question is asking and then look for the relevant information on the chart or graph.

Possible Instructional Activities

Graph Search

For homework, have students bring to class two different types of graphs (that is, bar, line, or picture graphs) from newspapers or magazines. Alternatively, have students create their own graphs using real or made-up data. For each graph, ask students to write a short paragraph that identifies the type of graph and briefly describes the data displayed.

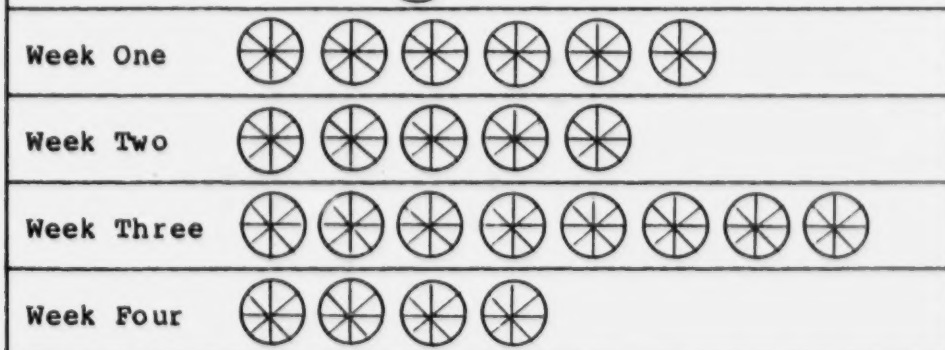
Select several of the graphs to discuss in class. Draw the graphs on the board or make copies for students to share. Ask questions that require students to perform computations using the information on the graphs.

Practice with Graphs

Give each student a copy of the problems on the following pages. Tell students to examine each graph carefully and answer the questions pertaining to each. Have students solve the problems.

**Number of Pizzas Sold by the School
During the Annual Pizza Sale Fund-Raiser**

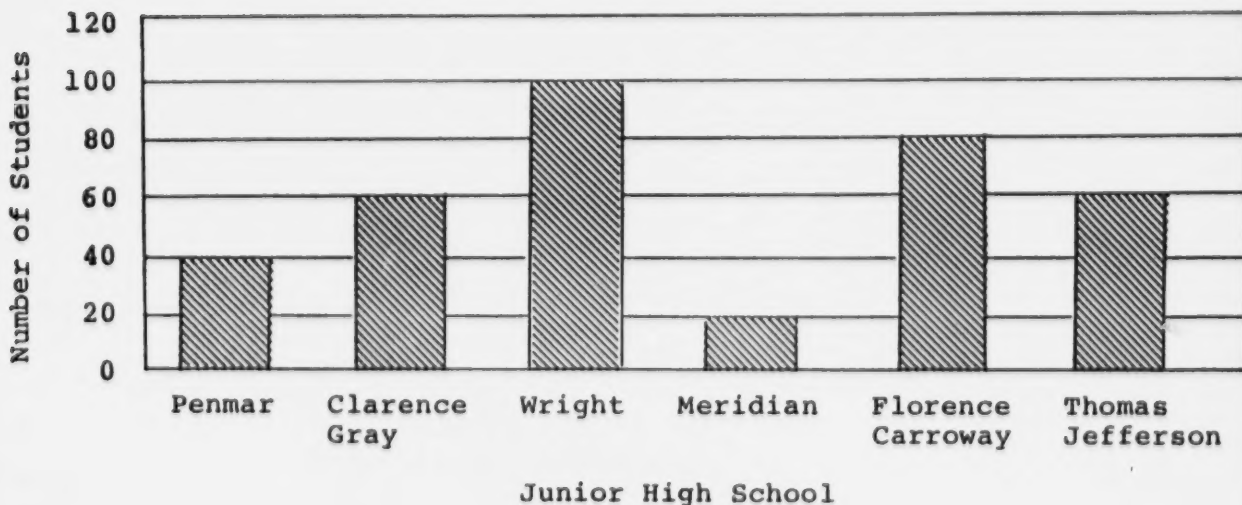
(Each  = 15 pizzas)



How many pizzas were sold during the first 2 weeks?

How many more pizzas were sold during week three than week four?

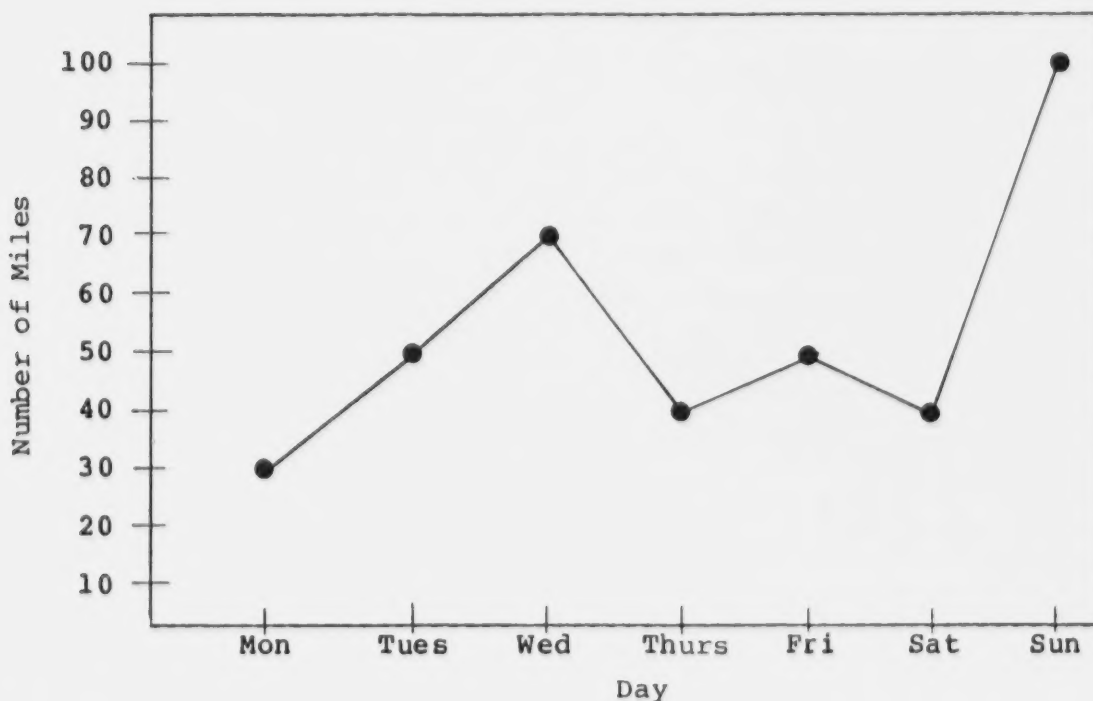
**Number of Students in School Choirs
at Six Junior High Schools**



How many students are in the school choirs at Penmar and Florence Carroway Junior High Schools? _____

How many more students are in the school choir at Thomas Jefferson Junior High School than at Clarence Gray Junior High School? _____

A Bicycle Racer's Log
of the Number of Miles Ridden Per Day



How many more miles did this bicycle racer ride on Tuesday than on Thursday? _____

How many miles were ridden on Wednesday and Thursday combined? _____

Population of Four Towns in Four Different Years

<u>Towns</u>	<u>1920</u>	<u>1940</u>	<u>1960</u>	<u>1980</u>
Kellyville	800	805	799	799
Copperton	5700	5680	5750	5780
Maytown	3565	3730	3885	4250
Peach Blossom	1800	1890	1910	2000

How many more people lived in Maytown in 1980 than in 1920?

What was the combined population of Kellyville and Peach Blossom in 1960? _____

Compare with Bars

Ask students to suggest possible topics for a bar graph to be constructed as a class activity. Select an appropriate topic that receives wide support from students. For example, students may choose to develop a bar graph to show how many students have a birthday in each month of the year. Either you or the students could do the actual construction of the graph. The completed graph can be displayed in the classroom. Ask students to solve a few computational questions that relate to the graph.

Objective 10: Determine the number of possible outcomes for a given situation.

This objective requires students to solve word problems involving possible outcomes for a given situation.

Sample Item

Mr. Jones has 2 suits, 5 shirts, and 2 ties. How many different combinations of one suit, one shirt, and one tie can he wear?

- A. 10 combinations
- B. 9 combinations
- C. 18 combinations
- D. 20 combinations

Test Questions

1. A test item will present a word problem that asks for the number of possible outcomes that can occur for a given situation.
2. Problem situations will involve clearly differentiated objects. The solution will not be dependent on the order of the objects.

Answer Choices

1. The correct answer choice will be the exact solution to the problem presented.
2. The number of possible outcomes specified in the answer choices will not exceed 24.
3. Incorrect answer choices are likely to result from one or more of the following types of errors:
 - a. Computation Error: An error in the multiplication of two numbers.
 - b. Incorrect Operation: Performing an operation other than the required one.
 - c. Regrouping Error: Failing to regroup when necessary, regrouping incorrectly, or regrouping unnecessarily.
 - d. Missing Operation: Not operating on the data or omitting one of the required operations.

Answer Analysis for Sample Item

1. Answer choice A is incorrect because of a missing operation. The answer 10 was obtained by multiplying 2 by 5 and failing to multiply again by the second 2.
2. Answer choice B is incorrect because the wrong operation was performed. The three numbers were added together rather than multiplied (that is, $2 + 5 + 2$ instead of $2 \times 5 \times 2$).
3. Answer choice C is incorrect because of a computation error. The product of 2 times 5 is 10, not 9, hence $2 \times 5 \times 2$ does not equal 18.
4. Answer choice D is the correct response. The product of $2 \times 5 \times 2$ is 20.

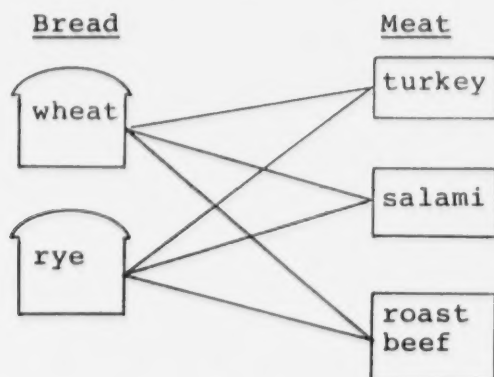
Instructional Emphases

To illustrate a "possible outcomes" situation to students, present the following example:

Suppose you have two kinds of bread (wheat and rye) and three kinds of meat (turkey, salami, and roast beef). How many different kinds of sandwiches could you make? In other words, what is the total number of possible ways to combine one kind of bread with one kind of meat?

To solve this problem, it is helpful to think of the breads and the meats as being separate sets of elements. Wheat and rye are the elements in the set of breads. Turkey, salami, and roast beef are the elements in the set of meats.

If we use a picture to represent each type of bread and each type of meat and draw a line connecting each bread with each meat, we can count the lines to determine that these elements can be combined in six different ways to make six different kinds of sandwiches. This counting procedure is illustrated below.



There is an easier way to determine the number of possible outcomes for two sets of factors: Multiply the number of elements in one set by the number of elements in the other set. Thus, two kinds of bread multiplied by three kinds of meat equals six possible combinations of one kind of bread with one kind of meat. Point out to students how much easier it is to solve this problem by multiplication than by representing all possible combinations and then counting them. Explain that the multiplication rule can be extended to situations involving three sets: Simply multiply the product of two sets of elements by the number of elements in the third set. To illustrate, suppose there are three kinds of bread, two kinds of meat, and four kinds of cheese. The number of possible combinations of one kind of bread, one meat, and one cheese is $(3 \times 2) \times 4$ or 24. Thus, for each additional set introduced in a situation, simply multiply the previous solution by the number of elements in the new set.

Possible Instructional Activities

Build a Menu

Have students create a fast food menu. Start with two food items and two beverages, such as the following:

<u>Food</u>	<u>Beverage</u>
hamburger	milk
pizza	lemonade

Based on this menu, ask students how many combinations of one food item with one beverage are possible. You may want to verify the correct answer ($2 \times 2 = 4$) by having students help you list each possible combination. With input from students, add items to either column of the menu, one item at a time. After each addition to the menu, ask students to use the multiplication method to calculate the new number of possible combinations.

Add another column, such as dessert, to demonstrate the effect of adding another set of elements to the problem.

Clothing Count

Tell students to imagine they are in charge of wardrobe coordination for a school play. For each character below, have students determine the number of possible combinations consisting of one of each kind of clothing or accessory specified.

CharacterCostume

Bob Slacking	1 sweater, 3 shirts, 2 pairs of pants
Cynthia Hedding	2 evening gowns, 4 purses
Charles Stomp	3 pairs of boots, 5 jackets
Lloyd Wynter	3 coats, 3 hats
Julie Nap	2 pairs of slippers, 3 pairs of pajamas, 2 robes

Unusual Outcomes

Have each student think of a situation and invent a corresponding problem that involves calculating the number of possible outcomes. Encourage students to be creative in developing their word problems. Keep in mind that the number of possible outcomes is limited to 24 in the TEAMS specifications. You may or may not want to apply this restriction to the problems that students create. Tell students to exchange papers and ask for volunteers to present and solve some of the more unusual problems before the class.

SKILL AREA: ALGEBRAIC CONCEPTS

This skill area of algebraic concepts consists of one objective, namely, solving one-step equations.

Objective 11: Solve one-step equations.

For this objective students will be presented with simple equations and asked to solve for the value of one variable. The solution will require one step.

Sample Item

Solve for n.

$$n - 26 = 5$$

- A. $n = 21$
- B. $n = 31$
- C. $n = 11$
- D. $n = 41$

Test Questions

1. A test item will require one addition, subtraction, or division operation to solve for the value of the variable.
2. The variable will appear on the left side of the equal sign.
3. Numbers for use in the equation will be positive whole numbers no larger than two digits.
4. The variable will be represented by the letter "n."
5. The equation will be written in one of three forms:
 - a. $n - 2 = 2$
 - b. $n + 2 = 4$
 - c. $2n = 4$

Answer Choices

1. Each answer choice will be a positive whole number. Answer choices will be expressed as follows: $n = [\text{a whole number}]$.

2. The correct answer choice will be the value of the variable that makes the equation true.
3. Incorrect answer choices are likely to result from one or more of the following types of errors:
 - a. Computation Error: An error in the addition, subtraction, multiplication, or division of two numbers.
 - b. Regrouping Error: Failing to regroup when necessary, regrouping incorrectly, or regrouping unnecessarily.
 - c. Incorrect Operation: Performing an operation other than the one required.

Answer Analysis for Sample Item

1. Answer choice A is incorrect because an incorrect operation was performed. The number 5 was subtracted from 26 to obtain 21.
2. Answer choice B is the correct response. To solve for n , 26 must be added to each side of the equation, as illustrated:

$$\begin{aligned} n - 26 &= 5 \\ n - 26 + 26 &= 5 + 26 \\ n &= 5 + 26 \\ n &= 31 \end{aligned}$$
3. Answer choice C is incorrect because of the same error made in answer choice A. In addition, a regrouping error was made in subtracting 5 from 26, resulting in 11 instead of 21.
4. Answer choice D is incorrect because of a regrouping error. In adding 26 to 5, it appears that 2 instead of 1 was carried to the tens place.

Instructional Emphases

To perform successfully in the area of algebraic concepts, students must feel confident in their ability to "manipulate" equations to solve for a variable. Emphasize that the variable must be set alone on the left side of the equation before it is possible to determine its value. Also, introduce the Properties of Equality as students will be required to use one of these properties to solve each equation:

Addition Property of Equality

Adding the same number to each side of an equation forms an equivalent equation. For example:

$$n - 3 = 10$$

and

$$n - 3 + 3 = 10 + 3$$

thus

$$n = 13$$

Subtraction Property of Equality

Subtracting the same number from each side of an equation forms an equivalent equation. For example:

$$n + 10 = 20$$

and

$$n + 10 - 10 = 20 - 10$$

thus

$$n = 10$$

Division Property of Equality

Dividing every term on each side of an equation by the same number forms an equivalent equation. For example:

$$6n = 24$$

and

$$\frac{6n}{6} = \frac{24}{6}$$

thus

$$n = 4$$

Be sure that students check their answer by substituting it into the equation.

Example:

$$n - 12 = 8$$

$$n = 12 + 8$$

$$n = 20$$

Check by substituting 20 for n.

$$20 - 12 = 8$$

$$8 = 8$$

To review, students should:

1. use the equality properties to set the variable alone on the left side of the equation,
2. do the necessary computations to solve for the variable, and
3. check the answer through substitution.

Possible Instructional Activities

Equations in Steps

This activity is intended to give students practice at working through the appropriate steps to solve algebraic equations. Tell students to solve for n in the following equations by filling in the blanks.

$$n + 28 = 36$$

$$n + 28 - \square = 36 - \square$$

$$n = \square$$

$$n + 22 = 51$$

$$n + 22 - \square = 51 - \square$$

$$n = \square$$

$$n - 14 = 47$$

$$n - 14 + \square = 47 + \square$$

$$n = \square$$

$$n - 13 = 72$$

$$n - 13 + \square = 72 + \square$$

$$n = \square$$

$$6n = 96$$

$$\frac{6n}{\square} = \frac{96}{\square}$$
$$n = \square$$

$$7n = 91$$

$$\frac{7n}{\square} = \frac{91}{\square}$$
$$n = \square$$

Algebra in Words

For each problem below have students identify the correct algebraic sentence represented and solve for x.

Example: The sum of a number x and 16 is 25.

$$x + 16 = 25$$

$$x + 16 - 16 = 25 - 16$$

$$x = 9$$

1. The number x added to 46 is 61.
2. The number x minus 67 is 17.
3. The product of 6 times the number x is 78.
4. The sum of the number x and 14 is 52.
5. The number 24 subtracted from the number x is 36.
6. The product of 4 times the number x is 56.
7. The number x added to 32 is 63.
8. The number x minus 25 is 31.
9. The product of 3 times the number x is 57.

Equation Code

Write the equations below on the board. Have students solve each equation, then find the letter in the key that corresponds to each solution. If each equation is solved correctly, the letters will form a familiar word.

Letter

$$n - 6 = 4 \quad \underline{\hspace{1cm}}$$

$$n + 8 = 25 \quad \underline{\hspace{1cm}}$$

$$n + 5 = 14 \quad \underline{\hspace{1cm}}$$

$$n - 17 = 36 \quad \underline{\hspace{1cm}}$$

$$3n = 42 \quad \underline{\hspace{1cm}}$$

$$n - 64 = 27 \quad \underline{\hspace{1cm}}$$

$$5n = 50 \quad \underline{\hspace{1cm}}$$

Key

9	10	14	15	22	17	53	91
G	A	B	X	Y	L	E	R